Implications of Overconfidence on Information Investment

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GT Lab Seminar
HSE St.Petersburg
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Is Overconfidence Bad or Good?

Forbes SEP 19, 2011 @ 11:29 AM 12,780

The Benefits and Danger of Over-Confidence

BBC capital

The trouble with being too confident

By Sydney Finkelstein
27 May 2015

TIME

CAREER STRATEGIES

You Gotta Have Faith: Why Overconfidence Can (Sometimes) Be Good for You

By David Futrelle | July 13, 2012

NATIONAL GEOGRAPHIC

Evolution of Narcissism: Why We're Overconfident, and Why It Works

Overestimating our abilities can be a strategy for success, model shows.

By **Christine Dell'Amore**, for National Geographic News

PUBLISHED SEPTEMBER 16, 2011

Outline

1. Model of *overconfidence as misperception of info precision* $\Rightarrow$ three forces that arise from overconfidence:
   $\uparrow$ overconfidence $\Rightarrow$ force 1 increases info investment, force 2 and 3 decrease it
2. Given the level of overconfidence, can we change the incentives to improve the outcome?

Literature Review

1. Overconfidence as correlation neglect:
   Ortoleva and Snowberg (2015), Levy and Razin (2015), Glaeser and Sunstein (2009)
2. Overconfidence as overesimation of one's ability:
   Heidhues, Koszegi and Strack (2015)
3. Overconfidence as overprecision, with no option to choose the amount of information to collect:
   Scheinkman and Xiong (2003), Kyle, Obizhaeva and Wang (2017)

This paper: overconfidence as overprecision, with the option to choose the amount of information to collect

PART 1. MODEL OF OVERCONFIDENCE

Leading example

A judge decides whether to acquit or convict a defendant who can be either innocent or guilty.

Research question

How does overconfidence influence the quality of the verdict?

Moore and Healy (2008): Three Types of Overconfidence

Overestimation of one's actual performance, *I did it great!*

Overplacement of one's performance relative to others, *I did it better than others!*

Overprecision in one's beliefs, *I know everything!*

Moore and Healy (2008): Three Types of Overconfidence

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This paper

The judge believes that he has access to information that is *more precise than it actually is*

- by consuming this information, he becomes overconfident in his beliefs $\longrightarrow$ *overprecision*
- by overestimating the precision of available information, he overestimates his ability to process this information $\longrightarrow$ *overestimation*

Model

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- Unbiased judge:
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- Info: Brownian motion with state-dependent drift

$$dX_t = \mu_z dt + \sigma dW_t, \quad \mu_z = \begin{cases} 1, & z = \text{Innocent} \\ -1, & z = \text{Guilty} \end{cases}$$

  - judge chooses the stopping time τ
  - cost = $\kappa \cdot \tau$

$$u(\text{verdict}, z) - \kappa\tau$$

The judge observes

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Definition

Overconfidence = distortion in perceived variance of the signals: the judge believes $\tilde{\sigma}^2$ instead of σ^2

$$\frac{\sigma^2}{\tilde{\sigma}^2} : \quad \text{the level of overconfidence}$$

Assumption: no variance updating

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Question

How does overconfidence influence the quality of the verdict?

or equivalently

How does the probability of the correct verdict change with the perceived variance $\tilde{\sigma}^2$?

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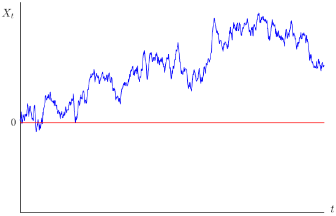
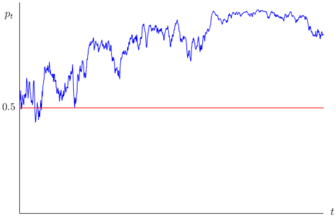
Optimal Strategy in Dynamic Model





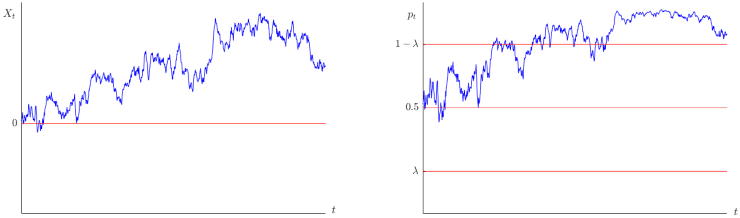
Optimal Strategy in Dynamic Model

Theorem

The optimal strategy exists and is given by

$$\tau = \inf\{t \geq 0 : \ p_t \in (\lambda, 1-\lambda)\}, \quad \text{verdict} = \begin{cases} \text{Acquit}, & p_\tau \geq 1-\lambda, \\ \text{Convict}, & p_\tau \leq \lambda. \end{cases}$$

where p_t is the belief at time t that $z = \text{Innocent}$.



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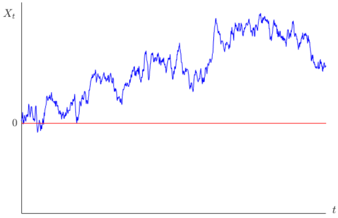

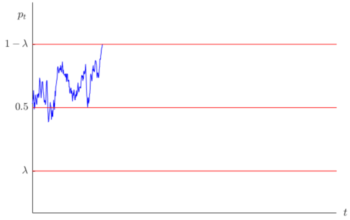

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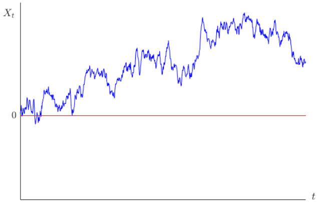
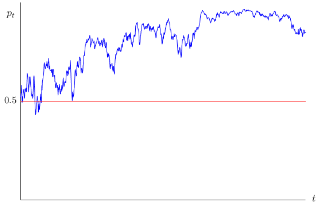


$\xrightarrow{\tilde{\sigma}}$



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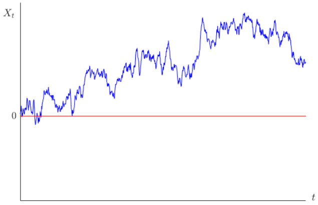

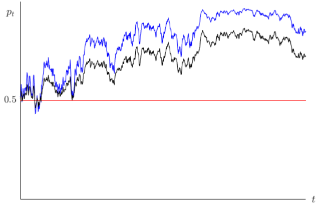

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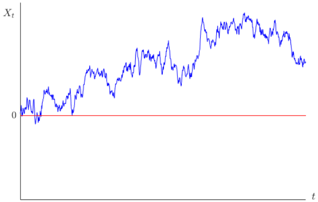

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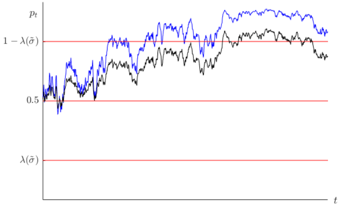

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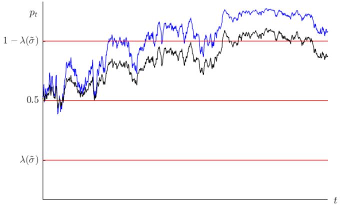

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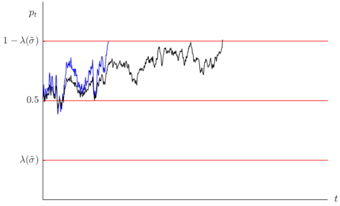

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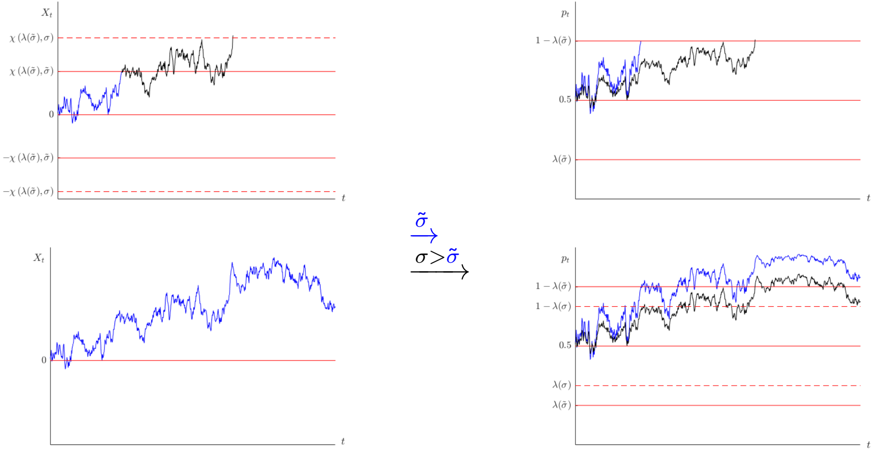

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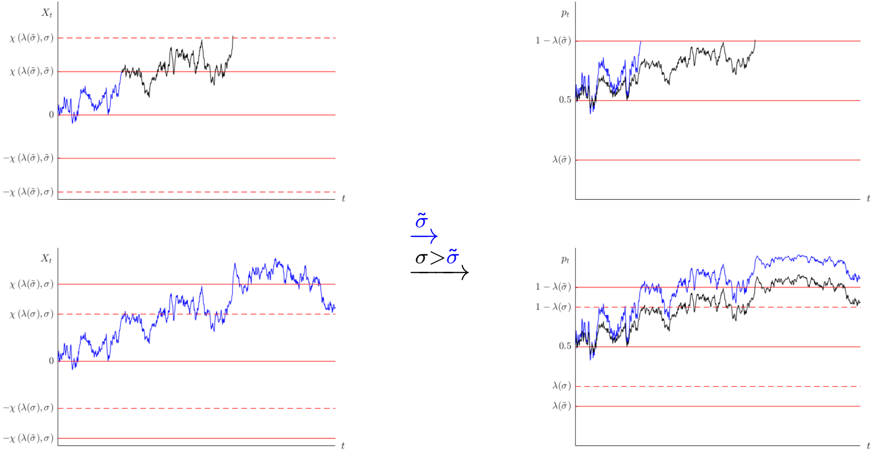

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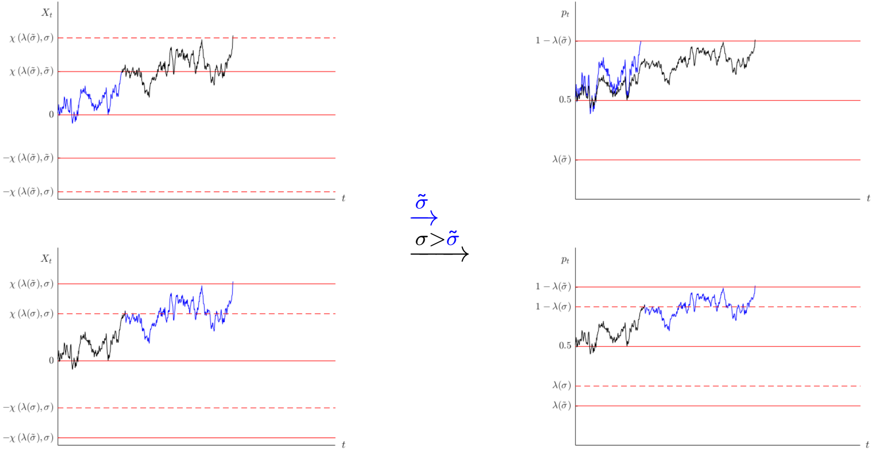

Overconfidence in Dynamic Model

Theorem
$\chi(\lambda(\sigma), \sigma)$ is increasing in σ

Direct effect : $\chi(\underbrace{\lambda(\sigma)}_{\text{keep standard of proof}}, \sigma\uparrow)$

Indirect effect : $\chi(\lambda(\sigma\uparrow)\downarrow, \underbrace{\sigma}_{\text{keep belief process}})$

An overconfident agent collects less information.

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Intuition?

Strategy space	Tradeoff	Result

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binary: $\tau \in \{0, T\}$ *Ex: hold trial or not*		
continuous: $\tau \geq 0$ *Ex: decide ex ante how long trial will be*		
function: choose τ dynamically *Ex: decide during trial when to stop it*		

Strategy space	Tradeoff	Result
binary: $\tau \in \{0, T\}$ *Ex: hold trial or not*	**Force 1** variance $\downarrow \Rightarrow$ signal is more precise $\Rightarrow$ WTP for signal $\uparrow \Rightarrow$ **more information**	overconfidence $\uparrow \Rightarrow$ $\mathbb{E}[\tau] \uparrow$

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continuous: $\tau \geq 0$ *Ex: decide ex ante how long trial will be*	**Force 1** **Force 2** variance $\downarrow \Rightarrow$ already collected info is more precise $\Rightarrow$ **less information**	

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continuous: $\tau \geq 0$ *Ex: decide ex ante how long trial will be*	**Force 1** **Force 2** variance $\downarrow \Rightarrow$ already collected info is more precise $\Rightarrow$ **less information**	$\exists$ optimal level of overconfidence $\mathbb{E}[\tau] \uparrow$ below it $\mathbb{E}[\tau] \downarrow$ above it

General Static Model

Timing:

1. Decide on the cost $c \geq 0$ of information
2. Collect information
   Assume the quality

$$h(c) := \underbrace{\frac{1}{2}\left(\text{Prob}(\text{Acquit}|\text{Innocent}) + \text{Prob}(\text{Convict}|\text{Guilty})\right)}_{\text{prob. of correct verdict}} - \frac{1}{2}$$

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Expected utility:

$$\left(\frac{1}{2} + h(c)\right) \underbrace{Q}_{\text{Utility(correct verdict)-R}} + \underbrace{R}_{\text{Utility(incorrect verdict)}} - c$$

General Static Model

$$\left(\frac{1}{2} + h(c)\right) Q + R - c$$

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$$c'(\eta) = -\frac{\overbrace{h'(\eta c)}^{\text{Force 1}} + \overbrace{\eta c h''(\eta c)}^{\text{Force 2}}}{\eta^2 h''(\eta c)}$$

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function: choose τ dynamically *Ex: decide during* *trial when to* *stop it*	**Force 1+Force 2** **Force 3** perceived variance $\downarrow \Rightarrow$ unexpected noise treated as meaningful signal $\Rightarrow$ stop sooner than expected $\Rightarrow$ **less information**	overconfidence $\uparrow \Rightarrow$ $\mathbb{E}[\tau] \downarrow$ *Force 3: excess sensitivity* *to noise* $\Rightarrow$ *strong when* *little info collected*

PART 2. OPTIMAL CONTRACT FOR OVERCONFIDENT AGENT

Assumption

The principal knows the level of overconfidence of the agent

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- contract = M: M$\in${dynamic model, static model}
  Should we restrict the judge to commit to the length of the trial in advance?
- contract = (M,Q): Q is the agent's payoff benefit from the correct verdict
  What if we can also choose how much to pay to the agent?

Dynamic vs Static Models

Goal: compare Prob(correct decision|dynamic model)$\equiv \Pi^D$ vs Prob(correct decision|static model)$\equiv \Pi^C$

- if the agent is rational, dynamic model is better
- dynamic model brings force 3 that decreases the probability of the correct decision

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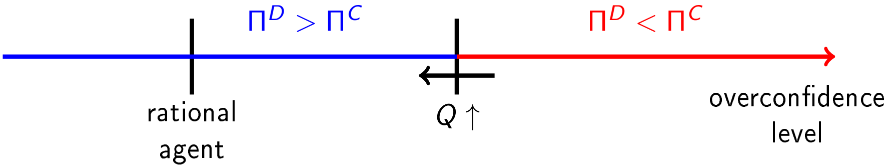
- if the agent is rational, dynamic model is better
- dynamic model brings force 3 that decreases the probability of the correct decision

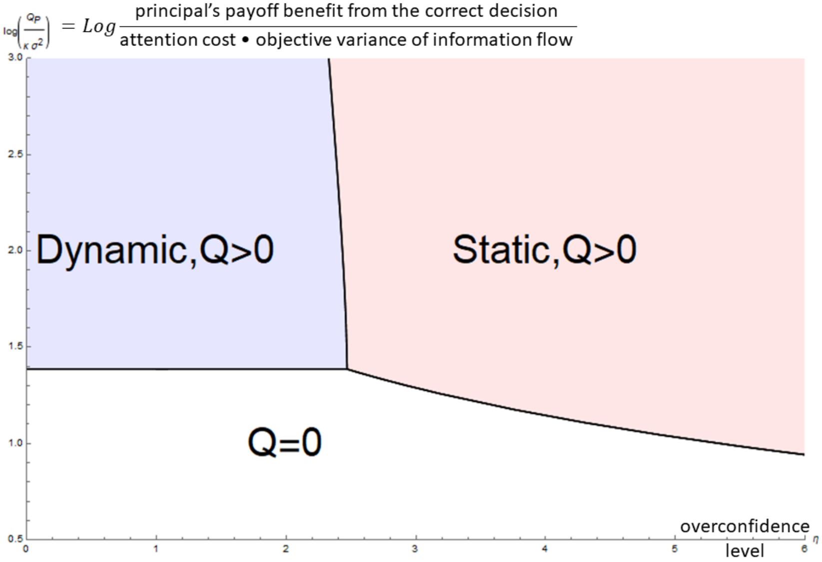
Theorem

There is a unique level of overconfidence such that $\Pi^D > \Pi^C$ below that level, and $\Pi^D < \Pi^C$ above it. Moreover, this level is decreasing in the agent's payoff benefit Q from the correct decision



- $Q \uparrow \Rightarrow$ for rational agent $\Pi^D \uparrow 1$ and $\Pi^C \uparrow 1$

Optimal Contract (Model, Q)



Conclusion

1. Model of overconfidence:
   - level of overconfidence = degree of misperception of information precision
   - $\uparrow$ overconfidence $\Rightarrow$

     force 1 : $\uparrow$ precision of the next piece of information $\Rightarrow$ more information

     force 2 : $\uparrow$ precision of already collected information $\Rightarrow$ less information

     force 3 : $\uparrow$ weight placed on noise when updating beliefs $\Rightarrow$ stop sooner than expected $\Rightarrow$ less information
2. Policy recommendation: force a highly overconfident decision maker to commit to the amount of information he is going to collect in advance