Dynamic Choice of Information Sources

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Why do we learn?

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“Physics is like sex: sure, it may give some practical results, but that's not why we do it.

Richard Feynman

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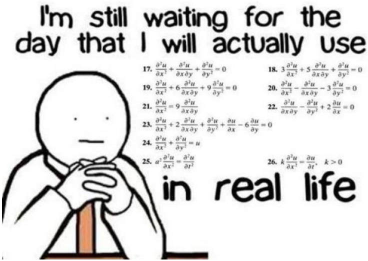

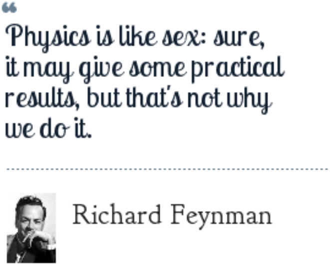

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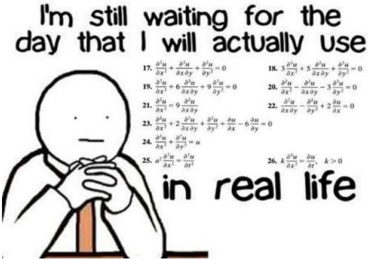
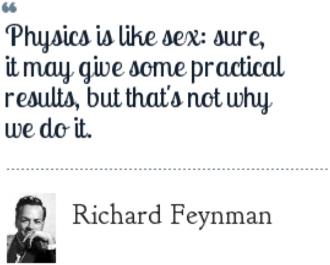


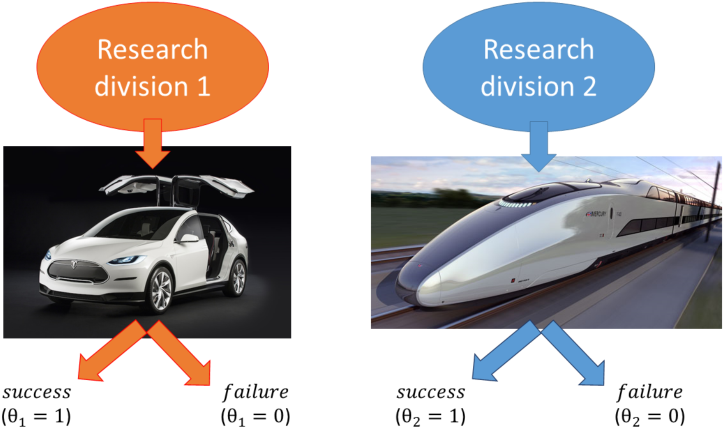


Motivation

Formulating the goal of learning is itself a hard task. *How much do we need to know about the goal to learn optimally?*

Example

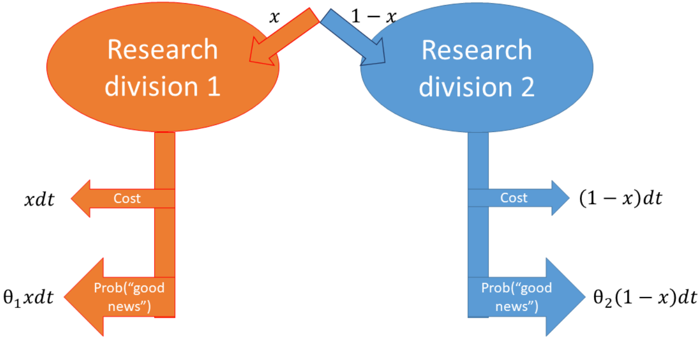
- A company has two research divisions
- Each division $k = 1, 2$ investigates the profitability of a certain project, which can be either success $(\theta_k = 1)$ or failure $(\theta_k = 0)$
  - θ_1 and θ_2 are not necessarily independent



Example

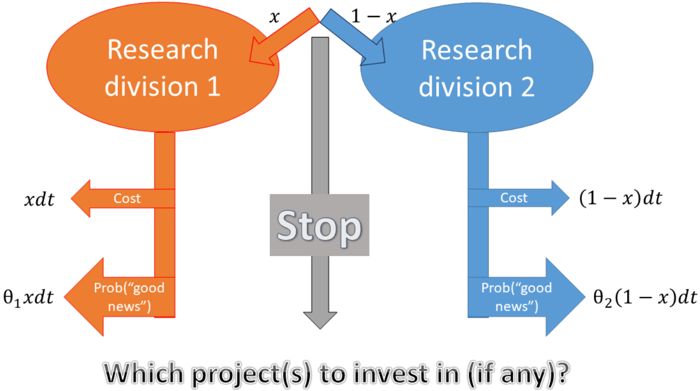
- At each instance of time, the company allocates a unit of resources between two divisions
- Each division k spends its resources $x_k \in [0,1]$ to search for a proof that its project is successful $\theta_k = 1$

	$\theta_k = 1$	$\theta_k = 0$
Prob("good news")	$x_k dt$	0
Prob("no news")	$1 - x_k dt$	1



Example

- The company decides when to stop research and make the decision



General Framework

Learning Stage

- four states of the world: $(\theta_1, \theta_2) \in \{(1,0), (0,1), (1,1), (0,0)\}$
- two information sources:
  - source 1 = search for a proof that $\theta_1 = 1$
  - source 2 = search for a proof that $\theta_2 = 1$

Decision Making Stage

- $\mathcal{A} = \{a_1, \ldots, a_n\}$: set of actions to choose from
- $u_i(a)$: payoff from choosing action a

	$(1,0)$	$(0,1)$	$(1,1)$	$(0,0)$
a_1	$u_1(a_1)$	$u_2(a_1)$	$u_3(a_1)$	$u_4(a_1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_n	$u_1(a_n)$	$u_2(a_n)$	$u_3(a_n)$	$u_4(a_n)$

General Framework

Learning Stage *How to learn?*

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a_n	$u_1(a_n)$	$u_2(a_n)$	$u_3(a_n)$	$u_4(a_n)$

Solution

Optimal strategy balances the trade-off:

- *belief-based incentives*: use the source that leads to a higher Prob("good news")
  - depends on current **beliefs**
- *payoff-based incentives*: learn about the project that leads to a higher profit if successful
  - depends on **payoff** matrix

Solution

All strategies can be divided into two classes:

1. No matter what happens in the future, at most one source is used.
2. Both sources can potentially be used (not necessarily simultaneously).

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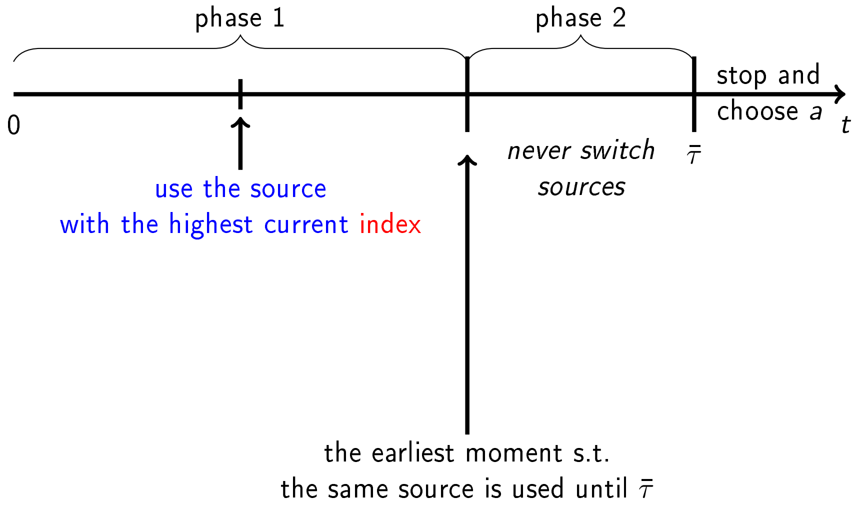
If the optimal strategy belongs to the second class, then

- choose the source with the highest index
- main result: expression for the index, how it depends on the payoff matrix

Optimal Strategy

In the absence of "good news"



Phase 1

Fix current beliefs
$(p_1 = Prob(\{1, 0\}), p_2 = Prob(\{0, 1\}), p_3 = Prob(\{1, 1\}))$.
If a positive signal occurs ***now****, what is the optimal strategy?*

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*If a positive signal occurs **now**, what is the optimal strategy?*

Four possibilities:

1. $\theta_1 = 1 \Rightarrow$ learn from source 2; $\theta_2 = 1 \Rightarrow$ learn from source 1
2. $\theta_1 = 1 \Rightarrow$ stop ; $\theta_2 = 1 \Rightarrow$ learn from source 1
3. $\theta_1 = 1 \Rightarrow$ learn from source 2; $\theta_2 = 1 \Rightarrow$ stop
4. $\theta_1 = 1 \Rightarrow$ stop ; $\theta_2 = 1 \Rightarrow$ stop

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Four possibilities:

1. $\theta_1 = 1 \Rightarrow$ learn from source 2; $\theta_2 = 1 \Rightarrow$ learn from source 1
   - phase 1 rule: indifferent between both sources
2. $\theta_1 = 1 \Rightarrow$ stop ; $\theta_2 = 1 \Rightarrow$ learn from source 1
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   - phase 1 rule: use source 1
3. $\theta_1 = 1 \Rightarrow$ learn from source 2; $\theta_2 = 1 \Rightarrow$ stop
   - phase 1 rule: use source 2
4. $\theta_1 = 1 \Rightarrow$ stop and choose a_1; $\theta_2 = 1 \Rightarrow$ stop and choose a_2
   - phase 1 rule: use the source with the highest index

Theorem

Phase 1 rule: use the source with the highest index

$$p_k + p_3 u_3(a_k)$$

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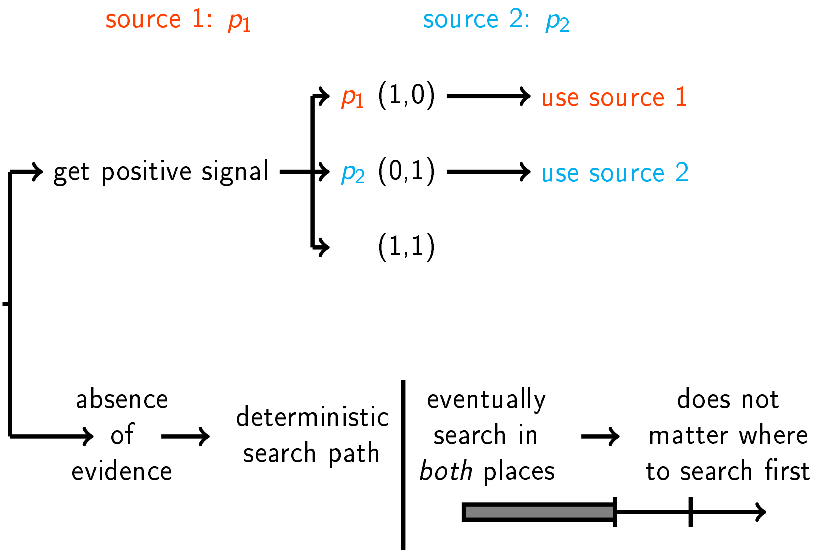
- get positive signal →
  - (1,0)
  - (0,1)
  - (1,1)
- absence of evidence → deterministic search path | eventually search in *both* places → does not matter where to search first

Theorem

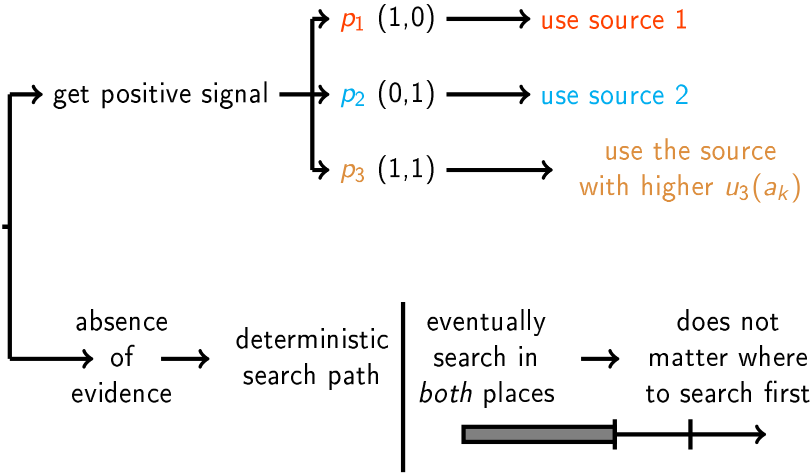
Phase 1 rule: use the source with the highest index



Theorem

Phase 1 rule: use the source with the highest index

source 1: $p_1 + p_3 u_3(a_1)$ source 2: $p_2 + p_3 u_3(a_2)$



Takeaway

The index depends only on the optimal actions in case of a breakthrough (state θ_k is revealed).

Why is it "good news"?

Examples:

- information sources = different diagnostic tests;
  decision problem also specifies possible treatments
  How much do we need to know about possible treatments to know which test to apply?

Takeaway

The index depends only on the optimal actions in case of a breakthrough (state θ_k is revealed).

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Examples:

- information sources = different diagnostic tests;
  decision problem also specifies possible treatments
  How much do we need to know about possible treatments to know which test to apply?
- city council decides on the fate of some area;
  this area is a critical habitat for a bird and a promising place for oil drilling;
  information sources = search for oil / a bird;
  problem: disagreement in the council
  All the council has to agree on is what to do if the bird or oil is found in another place

Literature

	Che&Mierendorff (2017, working paper)	Nikandrova&Pancs (2018, TE)	This paper
states	(1,0) and (0,1)		(1,0), (0,1), (1,1), (0,0)
correlation	$\theta_1 + \theta_2 = 1$	$\theta_1 \perp\!\!\!\perp \theta_2$	arbitrary
actions	2 ("match the state")	2 ("choose successful project")	any number
focus	on a given decision problem $\Downarrow$ optimal strategy (including chosen action)		on the general form of the learning process $\Downarrow$ index

Literature

Existing literature: actions and states are tightly connected

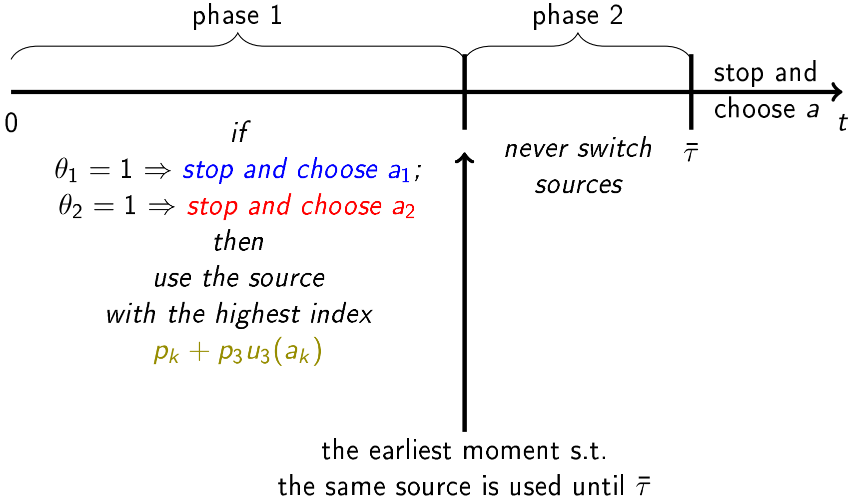
- Multi-armed bandit problem: actions $\sim$ states (info about actions directly, not through a state of the world)

  Pancs and Nikandrova 2018, Che and Mierendorff 2017, Klein and Rady 2011
- Search problem
  - minimize cost of learning conditional on finding the state

    Ahlswede&Wegener 1987
  - or finding the state has a direct consequences to the payoff (treasure hunt)

    Matros&Smirnov 2016, Fershtman&Rubinstein 1997
- *Liang, Mu, and Syrgkanis (2017)* came to the "opposite" conclusion that myoptic learning is (almost) optional
  - completely different structure of the decision problem; in particular, only one state component, θ_1, is payoff relevant

Optimal Strategy



Reminder: $p_1 = \mathbb{P}[(1,0)]$, $p_2 = \mathbb{P}[(0,1)]$, $p_3 = \mathbb{P}[(1,1)]$, u_3: payoff at (1,1)